Upward Seepage at the Downstream Toe of Hydraulic Structures External Suffosion / Fluidisation

Ahmed Khaddour

Institute of Forensic Engineering, Faculty of Civil Engineering, University of Technology Brno, Žižkova 17, 602 00 Brno, Czech Republic

Abstract: Embankment dams are made of earth and rock material and represent the most common dam type worldwide. One of the most common failure scenarios of embankment dams start with internal erosion. Problems associated with filtration and soil stability therefore play an important role in the design or rehabilitation of water structures. Filtration stability and deformation of soil due to seepage is reflected in the economic and safety assessment of these structures. On the subject of various aspects related to the stability and deformation of soils with respect to groundwater flow considerable number of works was developed worldwide.

Key words: External suffosion • Fluidisation • Critical hydraulic gradient

INTRODUCTION

External fluidization-Suffosion at Non Coherent Soil:
The upward seepage at the downstream toe of hydraulic structures such as dams or levees (Fig. 1) can result in the initiation of internal erosion of non-cohesive soils at the base. The theoretical analyses carried out by numerous authors have lead to the evaluation of the threshold velocity of filtration (specific discharge) and the hydraulic gradient as criteria for the initiation of the heave/external suffosion process.

Elementary volume upper right corner in (Fig. 1) is considered at the place of the outflow from a porous medium. The state of equilibrium is assumed then the sum of all vertical forces acting on the volume has to be equal to zero. Following expressions are using notation from (Fig. 2). Then the state of equilibrium can be expressed as \( \sum S = 0 \).

The “passive” (downward) vertical force consists of following components:

- Hydrostatic pressure force acting on the upper surface of the elementary volume... \( S_1 \)
- Weight of the porous medium - particles... \( \phi \)
- Weight of water in pores... \( S_w \)

Fig. 1: Flow under a dam including a detail of the upward flow, arranged according to [Budhu, Muni 2007].

Fig. 2: All vertical forces acting on at the elementary volume.

Corresponding Author: Ahmed Khaddour, Institute of Forensic Engineering, Faculty of Civil Engineering, University of Technology Brno, Žižkova 17, 602 00 Brno, Czech Republic.
E-mail: khaddour@seznam.cz.
The “active” (upward) vertical force $S_1$ is represented by hydrostatic pressure acting on the lower surface of the elementary volume [1],

Friction at the sides of the elementary volume is neglected in the analysis. The individual forces can be expressed as follows:

$S_1 = \gamma_s (h_1 - \Delta L - L_2) \cdot A$

$S_2 = \gamma_s \cdot \Delta L (1 - n) \cdot A$

$S_3 = \gamma_w \cdot n \cdot \Delta L \cdot A$

$S_4 = \gamma_s (h_2 - L_2) \cdot A$

where:

- $A$ Area of cross section of the cylinder [m$^2$] see (Fig. 2),
- $h_1$ Piezometric head below of soil sample [m],
- $h_2$ Piezometric head above of soil sample [m],
- $n$ Is soil porosity,
- $S_1$ Is hydrostatic pressure force,
- $S_2$ Is weight of the porous medium – particles,
- $S_3$ Is weight of water in pores,
- $S_4$ Is vertical force is represented by hydrostatic pressure,
- $L_1$ Is height of water above of the soil sample [m],
- $L_2$ Is height of water below of the soil sample [m],
- $\gamma_s$ Is the soil particle unit weight [N.m$^{-3}$],
- $\gamma_w$ Is the unit weight of water [N.m$^{-3}$],
- $\Delta L$ Is the height of soil in cylinder [m].

The equilibrium state yields:

$S_1 - S_3 - S_4 - S_4 = 0$

For equilibrium state at the outflow from a porous medium we define critical hydraulic gradient:

$$J_{cr} = \frac{h_1 - h_2}{\Delta L}$$

(1.1a)

When substituting from (1.1a) to (1.1b) we obtain well known Terzaghi formula

$$J_{cr} = \frac{(\gamma_s - \gamma_w) (1 - n)}{\gamma_s}$$

(1.1b)

where $J_{cr}$ is the critical hydraulic gradient. For porosity $n = 0.38$ and the solid phase unit weight $\gamma_s = 26$ kN/m$^3$, the critical value of hydraulic gradient would be approximately:

$J_{cr} \approx 1$

The results of Istomina’s research are frequently used in the practical assessment of the origin of heave/external suffosion in non-cohesive soils during upward seepage. According to Istomina, the critical hydraulic gradient $J_{cr}$ is expressed as a function of the non-uniformity coefficient $C_u$. However, Istomina’s laboratory results differ significantly in this respect and should be discussed in more detail [1],

In general, the resistance of non-coherent soil against heave/external suffosion is given by:

- The uniformity of the soil - the higher the uniformity, the higher the resistance,
- The compaction of the soil – the higher the compaction, the higher the shear strength and resistance of the soil,
- The shape of particles - round shaped grains are more susceptible to heave / suffosion.

Moreover, during laboratory investigations the methodology used is important; significant factors include the method of embedding the soil into the apparatus, the rate at which the hydraulic gradient is increased, the origin of local failure where the soil is in contact with the glass walls, etc.

Moreover, there are many times when it is extremely difficult to define the symptoms of the “failure,” as a sharp boundary mostly does not exist: firstly, very fine particles are flushed out from the sample, though this has practically no influence on the seepage velocity and hydraulic conductivity, the failure manifests itself both by the boiling of particles and the heaving (due to uplift UPL) of a significant part of the sample.

Some materials were tested for the $J_{cr}$ gradients identified at the moment of failure and for so called “steady” $J$ gradients maintained after the disturbance of the soil sample [2]. ‘Steady’ gradients were considered to exist due to the residual “resistivity” of a soil. Even if failure occurred, the soil samples were able to sustain a certain gradient $J$, though this was significantly lower then $J_{cr}$. Examples of the observed results are shown in (Table 1).

In order to guarantee “safe” design, Istomina firstly plotted the lower envelope of the observed critical gradients $J_{cr}$ as a function of the soil non-uniformity coefficient $C_u$. According to the requirements of today’s technical standards, such a defined $J_{cr}$ value should be considered as the so-called “standard” soil resistance.
Table 1: Resulting critical and “steady” hydraulic gradients [Istomina 1957]

<table>
<thead>
<tr>
<th>Material</th>
<th>Until failure</th>
<th>After failure</th>
<th>Gradients</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\sigma$ [g/cm$^3$]</td>
<td>porosity $\eta$</td>
<td>$d_0$ [mm]</td>
</tr>
<tr>
<td>Sand 3 – test 11</td>
<td>1.61</td>
<td>0.39</td>
<td>0.069</td>
</tr>
<tr>
<td>Sand 5 – test 38</td>
<td>1.70</td>
<td>0.36</td>
<td>0.117</td>
</tr>
<tr>
<td>Sand 4 – test 5</td>
<td>1.89</td>
<td>0.21</td>
<td>0.093</td>
</tr>
<tr>
<td>Sand 4 – test 44</td>
<td>1.87</td>
<td>0.29</td>
<td>0.083</td>
</tr>
<tr>
<td>Sand mixture 3 – test 36</td>
<td>1.79</td>
<td>0.32</td>
<td>0.065</td>
</tr>
<tr>
<td>Sand mixture 3 – test 43</td>
<td>1.85</td>
<td>0.3</td>
<td>0.093</td>
</tr>
</tbody>
</table>

Fig. 3: Plot of $J_{crit} = f(C_s)$ and $J_{per} = f(C_s)$ after [Istomina 1957].

Fig. 4: Example of situations where hydraulic fluidisation might be critical

Application on the External Suffosion/ Fluidisation Problems: When applying the (3.2) to the internal erosion - external suffosion, resp. and fluidisation - the load is represented by hydraulic gradient $J$, the resistance by the critical gradient $J_{crit}$. The typical situation of such failure is shown in (Fig. 4). The driving force for suffosion and fluidisation can be expressed via hydraulic gradient $J$.

Therefore, the limit state condition for the origin of external suffosion/fluidisation is based on comparison of hydraulic gradient $J$ in the soil with its critical value $J_{crit}$. Both hydraulic gradient $J$ and its critical value $J_{crit}$ suffer from significant uncertainty which should be taken into account during the assessment. In terms of limit state method this is taken into account via partial reliability factors (see below). According to the technical standards the following limit state relation can be proposed [3]:

$$\gamma_{fa} \cdot \gamma_p \cdot J_{crit} \leq \gamma_{fa} \cdot \gamma_{per} \cdot J_{crit}$$

where $\gamma_{fa}$ is performance factor, $\gamma_p$ is importance factor of the hydrotechnic structure, $J$ is local hydraulic gradient, $J_{crit}$ critical hydraulic gradient, $\gamma_{fa}$ is the load reliability factor given by the method of hydraulic gradient determination, e.g.:

- Using the in situ measurements
  $$\gamma_{fa} = 1.1,$$
- by hydraulic calculation
  $$\gamma_{fa} = 1.2,$$
- Estimated
  $$\gamma_{fa} \geq 1.3.$$
Preliminary Numerical Analysis of Seepage Flow:
To assess flow conditions in the vertical “Darcy” cylinder and to prescribe timing of experiments, preliminary numerical analysis has been done before the experimental research.

Formulation of the Problem: Vertical one-dimensional (1D) upward seepage flow (in the z direction) in the cylinder with the cross section area \( A \) filled with homogeneous material is described as follows [4]:

\[
k_s \frac{\partial^2 h}{\partial z^2} = S_0 \frac{\partial h}{\partial t}
\]  

(3.1)

where \( k_s \) is hydraulic conductivity, \( S_0 \) is specific storage.

In our case boundary conditions are expressing piezometric head at the entrance (lower) and outflow (upper) cross section of the cylinder. At the entrance section (\( z = z_0 \)) where piezometric head is changing during the test it holds:

\[
h(z = z_0, t) = h_1(t)
\]  

(3.2)

At the outflow section (\( z = z_2 \)) piezometric head is given by the overflow level at the outflow pipe:

\[
h(z = z_2, t) = h_2(t)
\]  

(3.3)

Initial Condition at the beginning of the test expresses hydrostatic no-flow conditions with

\[
h(z, t = 0) = h_0
\]  

(3.4)

where \( h_0 \) is level of the outflow pipe bottom (Fig. 5), \( h_1 \) piezometric head below of soil, \( h_2 \) piezometric head above of soil, \( z_0 \) the distance between datum level and below of soil sample and \( z_2 \) the distance between datum level and top of soil sample.

Numerical Solution: During the tests the question was how long one should wait after the boundary conditions change until the flow corresponds to the steady state. Therefore the preliminary numerical analysis was done. 1D simulation of the flow in cylindrical device was performed by the programme FILTR1D.EXE developed at the Institute of Water Structures FCE BUT. The code is based on the finite element method.

For the numerical test the dimension of the cylinder was identical with experimental device. The area of cylinder cross section is 0.011 m², the height of the sand sample is 0.20 m.

The analysis was carried out for the lowest identified permeability of the sample with hydraulic conductivity \( k = 5.10^{-6} \) m.s⁻¹. The compressibility modulus for the sandy sample was assumed to be 1 MPa. The initial conditions respected no-flow state at which \( h_1 = h_2 = 0 \) (the datum level chosen at the bottom of outflow pipe).

Boundary conditions were set according the relations (4.1) and (4.2). The datum level was located to the bottom of outflow pipe (Fig. 5). At the beginning of the test (\( t = 0 \)) the boundary conditions were \( h_1 = h_2 = 0 \). At the numerical test \( h_1 = 0.05 \) m was set for \( t > 0 \). Initial condition describe head at the beginning of the simulation (\( t = 0 \)). It was set \( h_0 = 0 \).

For the numerical solution the flow domain (0.2 m long sandy strata) was discretized into 20 finite elements each of the length 0.01 m. The size of the time step was tested and finally chosen 0.0001 hours, i.e. 0.36 s. The simulation was carried out until the flow reached approximately steady state conditions [1].

The results in terms of piezometric head in the apparatus at selected times is shown in (Fig. 6.).

Observation of Sample Defiance, Determination the Critical Hydraulic Gradient: The aim of observation is to determine the sample defiance, resp. breakdown. The definition of the instant of sample disruption is crucial in terms of the reading of critical hydraulic gradient. The available literature [2] does not distinguish
Fig. 6: The time which we need to have a steady state between piezometric head and height above the bottom of the sample.

Fig. 7: The disruption of the specimen starts with single “cracks” (heave) due to the uplift force UPL (Fig. 7) and finishes by complete total collapse (Fig. 8). [1].

Further on test of the influence of determination the disruption border was carried out as the part of the research. During the experiments No. 1, 2, 6, 7 and 8 the total collapse (Fig. 8) was considered as governing phenomena for the critical gradient reading, at the experiments 3 and 4 the initial flush out of fines was regarded as the critical state (Fig. 7). These led to significantly different results in assessed hydraulic gradients (see explanation below).

During the test the changing height of the sample and the changes in the colour or shape of the top layer of water cylinder was also observed. At the same time turbidity in water due to flushing out fine particles was visually monitored. The change in the soil structure was indicated also by the fluctuations of piezometric head measured in the apparatus. All these observations were recorded and stored in the computer, where all corresponding variables (piezometric head, hydraulic gradient, hydraulic conductivity) were evaluated and checked continuously during the test. It was found out that during the gradual collapse hydraulic conductivity increased considerably.

Such derived critical hydraulic gradients found out at each test were put in the database and subjected to statistical analysis.

**Comparison of Experimental Results with Values of Other Authors:**
The values of $J_{crit}$ derived by individual authors Terzaghi, Knorre, Zamarin, Pavlovski and Istomina are compared with results of our research in LWS. In (Fig. 9) the $J_{crit}$ values of individual authors are compared with the results of experimental research carried out at the LWS. Considerable differences are evident from the comparison. In practically all cases the theoretical $J_{crit}$ values are higher than those obtained in experimental research, where the value $J_{crit} = 1$ was only rarely exceeded. The shortcoming of the theoretical values is that they do not take into account soil non-uniformity, non-homogeneity and anisotropy. The significant difference in case of experiments 3 and 4 is due to the different indication of the instant of “failure,” which in this case was assumed at the moment of washout of fine particles from the sandy-gravel matrix and a more significant increase in hydraulic conductivity $k$. This occurred at relatively small hydraulic gradients i.e. $J_{crit} < 0.2$; nevertheless, there was practically no deformation of the sample (grain skeleton).
CONCLUSIONS

Five types of non-coherent soils with different soil properties were subject to the experimental research. Four soil types were transported from the field near the city of Brno, one was prepared artificially in the laboratory by mixing two types of different soils in prescribed proportion (1:1).

The research stated with the determination of the geotechnical properties of soils. Further on the investigation of the critical hydraulic gradient was carried out within 8 experiments consisting of total amount 321 single tests. The resulting critical hydraulic gradients were subjected to the statistical analysis; the results were compared with the investigations of other authors. The analysis of the determination of the instant of the sample defiance was the part of the research [3].

It can be stated that the results of our analysis in experiments 1, 2, 5, 6, 7, 8 in terms of 5% percentile of critical hydraulic gradients (which can be regarded as the “characteristic value” are comparable with values derived by Terzaghi, Knorre and Zamarin. These results are also compatible with the results of Istomina [2] where the startup process of critical hydraulic gradient is the appearance of cracks in the compaction samples and boiling of the fine particles in the no compaction samples. Results were not compatible in experiments 3 and 4 due to the different chosen symptoms of sample deterioration.

Laboratory results in experiments 1, 2, 5, 6, 7, 8 indicate good agreement with Istomina’s conclusions for uniformity coefficient of soils $C_u < 5$ (experiments 1, 6, 7), where the moment of failure due to heave, internal erosion is sharp and evident. In non-uniform soils the results are much more different and are strongly dependent on the definition and identification of the failure instant. When the washout of fine particles from the sample is the criterion, experiments show $J_{crit}$ values which are considerably smaller than in Istomina’s envelope curve (see the results for experiments 3, 4. When the failure criteria correspond to an overall collapse of the soil matrix, obtained critical gradients $J_{crit}$ practically in all cases exceed the expected limit (see the results for $C_u < 27.32$ (experiments 8), this is especially true in the case of compacted soils.

The most important conclusion is that the research proved validity of the results from old research [Istomina 1957]... and that the characteristic value of critical hydraulic gradient can be taken from the envelope curve for given uniformity coefficient.

Further conclusions and recommendations coming from the research are as follows:

- The minimum recommended trials for statistical assessment is about 30, in our analysis about 40 tests were carried out at each experiment for each soil sample. Our recommendation based on experience from the testing is that optimum number of tests should be between 60 to 80.
- Our analysis proved log-normal probability density function as the best-fit distribution function for the measured values of critical hydraulic gradient.
- For the reading of critical hydraulic gradient the selection the instant of sample disruption is crucial. Our recommendation is that the flush-out of finest particle should not be considered as sample defiance. The instant of the sample disruption should be related to the movement of particles belonging to soil skeleton which consists of coarser grains.
- It would be desirable to continue with the systematic measurements of critical hydraulic gradients for different soils both compacted and loose.
REFERENCES